Effects of plasma flows on particle diffusion in stochastic magnetic fields

M. Vlad,[1,*] F. Spineanu,[1,*] J. H. Misguich,[1] and R. Balescu[2]
[1]Association Euratom-Commissariat à l'Energie Atomique sur la Fusion, Département de Recherche sur la Fusion Contro1ée, Centre d'Etudes de Cadarache, 13108 Saint-Paul-lez-Durance Cedex, France
[2]Association Euratom-Etat Belge sur la Fusion, Physique Statistique et Plasmas, CP 231, Université Libre de Bruxelles, Campus Plaine Boulevard du Triomphe, 1050 Bruxelles, Belgium

(Received 18 December 1995; revised manuscript received 1 March 1996)

The study of collisional test particle diffusion in stochastic magnetic fields is extended to include the effects of the macroscopic flows of the plasma (drifts). We show that a substantial amplification of the diffusion coefficient can be obtained. This effect is produced by the combined action of the parallel collisional velocity and of the average drifts. The perpendicular collisional velocity influences the effective diffusion only in the limit of small average drifts. [S1063-651X(96)01807-7]

PACS number(s): 52.25.Fi, 05.40.+j, 52.25.Gj, 52.35.Ra

I. INTRODUCTION

The problem of collisional particle diffusion in stochastic magnetic fields, although not entirely solved, seems to be better understood at this moment. The early results of Rechester and Rosenbluth [1] and Kadomtsev and Pogutse [2] are now confirmed by more elaborated studies [3–12] and by numerical calculations [13]. The general result is that the stochastization of the magnetic lines could provide a possible explanation for the anomalous transport in magnetically confined plasmas. In this context we present here a study of the effects of the plasma flows on the transport coefficients. Such flows appear as deterministic drifts superposed on the stochastic velocity which determines collisional particle motion in a stochastic magnetic field. They can be produced by an electric field perpendicular to the confining magnetic field, by a current along it, by fast neutral beam injection, etc. Such macroscopic plasma flows are commonly found in experiments. We have obtained a nontrivial dependence of the effective particle diffusion coefficient on the deterministic drift velocity: first there is a strong increase of the diffusion which is followed, at higher values of the drift velocity, by a reversed effect, the increase of the drift leading to the decay of the diffusion. This happens for drift velocities both perpendicular and parallel to the main magnetic field, but in the case of a parallel drift the effect is superposed on the diffusion determined by the average motion along magnetic field lines (which is linear in the velocity). The effect of the deterministic drifts is important in all diffusion regimes. We have shown that the collisional cross field diffusivity $\chi_\perp$ has a significant influence on the effective diffusion coefficient only in the limit of small drift velocities; at higher values of these velocities, the diffusion coefficient is practically independent of $\chi_\perp$.

This problem of average drifts was not much analyzed in the literature. It was probably expected that the deterministic drifts determine, as in several related problems, a decrease of the diffusion coefficient so that they do not contribute to explaining the anomalous transport. To our knowledge, there are only the papers by Mynick and Krommes [14], by Myra and co-workers [15,16], by Coronado, Vitela, and Akcasu [17], and recently by Isichenko and Diamond [18] which deal with this problem but only in the case of noncollisional particles (e.g., runaway electrons). All these results show an attenuation of the diffusion due to deterministic drifts. Such behavior appears when the Lagrangian two point correlation of the stochastic velocity is a positive function at all times. We show here that in the doubly stochastic process, which determines the evolution of the collisional particles in the stochastic magnetic field, the effect of the deterministic drifts is more complex, precisely due to the fact that the Lagrangian correlation function of the stochastic velocity has a negative tail. It determines an increase of the diffusion coefficient with the drift velocity up to a maximum value which can be much larger than the driftless one. A similar peaking of the particle diffusion coefficient was also obtained in Ref. [19] from a qualitative analysis of the transport processes in the ergodic divertors of tokamak devices.

The present work belongs to a series of papers [7,10–12,20] which is devoted to various aspects of the magnetic turbulence. We use here the methods and some of the results obtained in our previous paper [7].

The text is organized as follows. The model is presented in Sec. II. It is a test particle approach based on Langevin-type equations. The comparison with the kinetic approach has been discussed in Ref. [10]. The diffusion coefficient as a function of the drift velocities is determined in Sec. III for a zero cross field collisional diffusivity. Next, in Sec. IV, the influence of the cross field collisional diffusivity is estimated. The conclusions are summarized in Sec. V.

II. THE MODEL

We consider the simplest geometry of the unperturbed magnetic configuration consisting of a constant confining magnetic field $\mathbf{B}_0$ (shearless slab model). The particle trajectories (in the Cartesian coordinates with the z axis along $\mathbf{B}_0$),

*On leave of absence from the Institute of Atomic Physics, P.O. Box MG-7, Magurele, Bucharest, Romania.

$\mathbf{x}(t)\equiv(x(t),y(t),z(t))$ are described by the following set of Langevin equations:

$$\frac{d\mathbf{x}_\perp}{dt}=\mathbf{b}(\mathbf{x})\,\frac{dz}{dt}+\boldsymbol{\eta}_\perp(t)+\mathbf{u}_\perp\,, \tag{2.1}$$

$$\frac{dz}{dt}=\eta_\parallel(t)+u_\parallel\,, \tag{2.2}$$

where $\mathbf{x}_\perp(t)\equiv(x(t),y(t))$, $\mathbf{b}=(b_x\,(\mathbf{x}),b_y(\mathbf{x}))$ is the stochastic perturbation of the magnetic field adimensionalized with B_0, $\eta_\parallel(t)$, $\boldsymbol{\eta}_\perp(t)$ are the components of the stochastic collisional velocity parallel and, respectively, perpendicular to $\mathbf{B}_0$ and $u_\parallel$, and $\mathbf{u}_\perp\equiv(0,u_\perp)$ are deterministic (average) components of particle velocity (the y axis was taken for simplicity along the velocity $\mathbf{u}_\perp$). Such deterministic motion can be produced, e.g., by a current along $\mathbf{B}_0$ or, respectively, by an electric field in the x axis direction (a radial electric field in tokamak plasmas). The magnetic drifts specific to tokamak configurations (which cannot be introduced in this model) have been extensively studied in Refs. [14–16].

Concerning the statistical properties of the random quantities we make the following reasonable assumptions. The collisional velocity has zero average and is modeled by Gaussian colored noise with the two time correlation function having different amplitudes in the parallel and perpendicular directions:

$$\langle\,\eta_\parallel(t)\,\eta_\parallel(t')\rangle=\chi_\parallel\nu\,\exp(-\nu|t-t'|)\equiv R_\parallel(|t-t'|), \tag{2.3}$$

$$\langle\,\eta_\perp^x(t)\,\eta_\perp^x(t')\rangle=\langle\,\eta_\perp^y(t)\,\eta_\perp^y(t')\rangle=\chi_\perp\nu\,\exp(-\nu|t-t'|)\equiv R_\perp(|t-t'|), \tag{2.4}$$

$$\langle\,\eta_\perp^x(t)\,\eta_\perp^y(t')\rangle=\langle\,\eta_\parallel(t)\,\eta_\perp^x(t')\rangle=\langle\,\eta_\parallel(t)\,\eta_\perp^y(t')\rangle=0, \tag{2.5}$$

where ν is the collision frequency of the plasma, $\chi_\parallel=V_T^2/(2\nu)$ is the (classical) parallel diffusion coefficient, $\chi_\perp=(V_T^2\nu)/(2\Omega^2)$ is the collisional cross field diffusion coefficient, and V_T is the thermal velocity.

For describing the stochastic magnetic field we introduce the vector potential $\Psi(\mathbf{x})\mathbf{e}_z$ in order to have the zero divergence condition ensured. $\Psi(\mathbf{x})$ is taken as a Gaussian random field, spatially homogeneous and isotropic in the (x,y) plane with an Eulerian autocorrelation function modeled by

$$\mathcal{A}(\mathbf{r})\equiv\langle\Psi(\mathbf{x}+\mathbf{r})\Psi(\mathbf{x})\rangle=\beta^2\lambda_\perp^2\exp\left(-\frac{r_z^2}{2\lambda_\parallel^2}-\frac{r_\perp^2}{2\lambda_\perp^2}\right). \tag{2.6}$$

Here, three characteristic parameters are defined: the parallel correlation length $\lambda_\parallel$, the perpendicular correlation length $\lambda_\perp$, and the dimensionless measure of the intensity of the stochastic magnetic field β. r_z and $r_\perp\equiv(r_x,r_y)$ are the components of the distance $\mathbf{r}$ between the two points. The Eulerian autocorrelation function of the potential $\Psi(\mathbf{x})$ determines the Eulerian correlations for the magnetic field components [7]:

$$\mathcal{B}_{ij}(\mathbf{r})\equiv\langle b_i(\mathbf{x}+\mathbf{r})b_j(\mathbf{x})\rangle=\frac{\mathcal{A}(r)}{\lambda_\perp^2}\left[\delta_{ij}-\frac{r_\perp^2\delta_{ij}-r_ir_j}{\lambda_\perp^2}\right]. \tag{2.7}$$

The correlation functions correspond in the Fourier representation to the wave number spectrum of the fluctuations of the magnetic field components and Eq. (2.7) is equivalent with the following model for the latter (see Ref. [7]):

$$\langle b_i(\mathbf{k})b_j(\mathbf{k})\rangle=(k_\perp^2\,\delta_{ij}-k_ik_j)\mathcal{A}(\mathbf{k}), \tag{2.8}$$

$$\mathcal{A}(\mathbf{k})=(2\pi)^{-3/2}\lambda_\parallel\lambda_\perp^4\beta^2\exp(-\tfrac{1}{2}\lambda_\parallel^2k_\parallel^2-\tfrac{1}{2}\lambda_\perp^2k_\perp^2), \tag{2.9}$$

where $k_\perp=\sqrt{k_x^2+k_y^2}$, $k_\parallel=k_z$, and $\mathcal{A}(\mathbf{k})$ is the spectrum of the fluctuating potential $\Psi(\mathbf{x})$. The factor contained in the parentheses on the right-hand side of Eq. (2.8) accounts for the zero divergence of the stochastic magnetic field.

Even in the absence of the deterministic drifts the problem of collisional particle evolution in a stochastic magnetic field is very complicated and it was not possible to find until now a simple and complete solution. We develop here a simplified model which is valid for magnetic perturbations such that $\alpha\equiv\beta(\lambda_\parallel/\lambda_\perp)\ll 1$ (quasilinear limit). In dealing with this model, we first neglect the perpendicular collisional stochastic velocity in Eq. (2.1) [$\boldsymbol{\eta}_\perp(t)=0$] and show that the problem can be solved in these conditions. The diffusion coefficient is obtained as a function of the drift velocities $u_\perp$, $u_\parallel$ (Sec. III). Although the perpendicular collisional stochastic velocity $\boldsymbol{\eta}_\perp(t)$ entails a strong mathematical complication of the problem which prevents the use of the method presented before, it is possible to make an approximate evaluation of the effect of the perpendicular collisional diffusivity on the effective diffusion coefficient. It is based on the calculation of the time of decorrelation induced by collisions (Sec. IV).

III. DRIFT INDUCED DIFFUSION COEFFICIENT

Equations (2.1) and (2.2) determine the particle trajectory $\mathbf{x}_\perp(t)$ which is actually a function of two variables: $\mathbf{x}_\perp(t)=\mathbf{x}_\perp(z(t),t)$, where the dependence on z results from the first term on the right-hand side of Eq. (2.1), the explicit dependence on time from the last two terms, and $z(t)$ is the solution of Eq. (2.2).

We consider in this section $\boldsymbol{\eta}_\perp(t)=0$ in Eq. (2.1). Consequently, the explicit dependence on time in the equation of motion is deterministic. We show here that this particular problem can be solved in the quasilinear limit characterized by $\alpha\equiv\beta(\lambda_\parallel/\lambda_\perp)\ll 1$. The method is similar to that developed for the driftless case in our previous work [7].

The mean square displacement of the trajectories in the direction i, averaged over the fluctuating magnetic field in a given realization of the stochastic velocity $\eta_\parallel(t)$, is calculated as

$$\langle[\mathbf{x}_\perp(t)-\mathbf{x}_\perp(0)-\mathbf{u}_\perp t]_i^2\rangle_b=\int_0^t\int dt_1dt_2\mathcal{L}_{ii}^{u_\perp}(z(t_1)-z(t_2),t_1-t_2)\,\frac{dz}{dt_1}\,\frac{dz}{dt_2}, \tag{3.1}$$

where $z(t)$ is considered as a given function of time, $\langle\,\rangle_b$ denotes the statistical average over the stochastic parameter specified by the subscript, and $\mathcal{L}_{ij}^{u_\perp}$ is the two point Lagrangian correlation of b_i, b_j along particle trajectories. It is related to the corresponding Eulerian correlation $\mathcal{B}_{ij}$ in the well-known Corrsin approximation [21] (valid in the quasilinear limit $\alpha \ll 1$):

$$\mathcal{L}_{ij}^{u_\perp}(\zeta,t) \equiv \langle b_i(\mathbf{x}_\perp(\zeta,t),\zeta) b_j(0,0)\rangle_b = \int_{-\infty}^{\infty}\int dx\, dy\, \mathcal{B}_{ij}(x,y,\zeta)\gamma(x,y;\zeta,t). \tag{3.2}$$

Here, $\gamma(x,y;\zeta,t)$ is the probability density for a displacement x,y of the trajectories which advance a distance ζ in a time interval t [$\zeta = z(t)$]. This probability is estimated as the average over the stochastic magnetic field of the Dirac δ function (see Ref. [7]):

$$\gamma(x,y;\zeta,t) = \langle \delta(x - x(\zeta,t))\delta(y - y(\zeta,t))\rangle_b. \tag{3.3}$$

Using the Fourier representation of the δ functions and performing the average of the resulting exponential by means of the cumulant method (second cumulant), one obtains

$$\gamma(x,y;\zeta,t) = \frac{1}{2\pi}\frac{1}{\sqrt{q}}\exp\left[-\frac{x^2 I_{yy} + (y - u_\perp t)^2 I_{xx} - 2x(y - u_\perp t) I_{xy}}{q}\right], \tag{3.4}$$

where $I_{ij}(\zeta,t) = \langle[\mathbf{x}_\perp(t) - \mathbf{x}_\perp(0) - \mathbf{u}_\perp t]_i [\mathbf{x}_\perp(t) - \mathbf{x}_\perp(0) - \mathbf{u}_\perp t]_j\rangle_b$ and $q(\zeta,t) = I_{xx} I_{yy} - I_{xy}^2$. The functions I_{ij} depend on the Lagrangian correlations $\mathcal{L}_{ij}^{u_\perp}$ according to Eq. (3.1) (and to the similar equations for the other components of the trajectory). Corrsin approximation (3.2) and the Eulerian correlations (2.7) determine a rather complicated set of coupled nonlinear integral equations for $\mathcal{L}_{ij}^{u_\perp}(\zeta,t)$. The equation for $\mathcal{L}_{xy}^{u_\perp}$ is homogeneous and as $\mathcal{L}_{xy}^{u_\perp}(0,0) = \mathcal{B}_{xy}(0,0,0) = 0$, its solution shows that the cross correlation remains zero, $\mathcal{L}_{xy}^{u_\perp}(\zeta,t) = 0$. The other two equations become

$$\mathcal{L}_{xx}^{u_\perp}(\zeta,t) = \beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right)\frac{\lambda_\perp^4 (J_y - u_\perp^2 t^2)}{J_x^{1/2} J_y^{5/2}}\exp\left[-\frac{u_\perp^2 t^2}{2 J_y}\right], \tag{3.5}$$

$$\mathcal{L}_{yy}^{u_\perp}(\zeta,t) = \beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right)\frac{\lambda_\perp^4}{J_x^{3/2} J_y^{1/2}}\exp\left[-\frac{u_\perp^2 t^2}{2 J_y}\right], \tag{3.6}$$

where $J_i \equiv \lambda_\perp^2 + I_{ii}$, $i = x,y$.

We note that the perpendicular drift motion along y has two important consequences. It destroys the gyrotropic property of the problem leading to a dissymmetry between x and y directions. Consequently, the Lagrangian correlations lose the symmetry property found in Ref. [7] and the diffusion coefficient becomes a (diagonal) tensorial quantity. On the other hand, when $u_\perp \neq 0$ it is not possible to perform the change of variables $t_n \to \xi_n = z(t_n)$, $n = 1,2$ in the integrals in Eq. (3.1) and thus I_{ii} are not only functions of $\zeta = z(t)$ and t but they depend on the whole trajectory $z(t_1)$, $t_1 \in [0,t]$. Due to this fact, Eqs. (3.5) and (3.6) are much more complicated than their driftless limit [Eq. (30) in Ref. [7]]. However, for $\alpha \ll 1$ (quasilinear limit), the nonlinearity consisting in the dependence of J_i on the unknown functions $\mathcal{L}_{ii}^{u_\perp}$ can be neglected (as shown in Ref. [7]) and Eqs. (3.5) and (3.6) reduce to simple approximate expressions for the Lagrangian correlations of the magnetic field components:

$$\mathcal{L}_{xx}^{u_\perp}(\zeta,t) = \beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right)\exp\left[-\frac{u_\perp^2 t^2}{2\lambda_\perp^2}\right]\left[1 - \frac{u_\perp^2 t^2}{\lambda_\perp^2}\right], \tag{3.7}$$

$$\mathcal{L}_{yy}^{u_\perp}(\zeta,t) = \beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right)\exp\left[-\frac{u_\perp^2 t^2}{2\lambda_\perp^2}\right]. \tag{3.8}$$

In these conditions, the average over the second random parameter, the collisional parallel velocity $\eta_\parallel(t)$ can be easily performed. The mean square displacement averaged over the two stochastic functions becomes

$$\Gamma_i(t) \equiv \langle[\mathbf{x}_\perp(t) - \mathbf{x}_\perp(0) - \mathbf{u}_\perp t]_i^2\rangle_{b\parallel} = \int_0^t\int dt_1 dt_2 L_{v_i}(t_1 - t_2; u_\parallel, u_\perp), \tag{3.9}$$

where $L_{v_i}(\tau; u_\parallel, u_\perp)$ is the Lagrangian two point correlation of $v_i \equiv b_{\mathbf{i}}(dz/dt)$, the perpendicular velocity determined by particle motion along *perturbed* magnetic lines:

$$L_{v_i}(t_1 - t_2; u_\parallel, u_\perp) \equiv \left\langle \mathcal{L}_{ii}^{u_\perp}(z(t_1) - z(t_2), t_1 - t_2)\frac{dz}{dt_1}\frac{dz}{dt_2}\right\rangle_\parallel. \tag{3.10}$$

These correlations can be calculated by performing a Fourier expansion of $\mathcal{L}_{ii}^{u_\perp}$ in the z argument, which leads to an average of the type

$$\left\langle \exp\{-ik[z(t_1) - z(t_2)]\}\frac{dz}{dt_1}\frac{dz}{dt_2}\right\rangle_\parallel = \frac{1}{k^2}\frac{\partial}{\partial t_1}\frac{\partial}{\partial t_2}\langle \exp\{-ik[z(t_1) - z(t_2)]\}\rangle_\parallel, \tag{3.11}$$

where the average on the right-hand side is determined using the cumulant theorem (second cumulant). After integration over the Fourier conjugate argument, one gets

$$L_{v_i}(\tau; u_\parallel, u_\perp) = \frac{1}{\sqrt{2\pi\langle\langle z^2(\tau)\rangle\rangle}}\int_{-\infty}^{\infty} d\zeta\, \mathcal{L}_{ii}^{u_\perp}(\zeta,\tau) \times \exp\left(-\frac{(\zeta - u_\parallel\tau)^2}{2\langle\langle z^2(\tau)\rangle\rangle}\right)\left[R_\parallel(\tau) - \frac{\varphi^2(\tau)}{\langle\langle z^2(\tau)\rangle\rangle} + \left(u_\parallel + \frac{\varphi(\tau)(\zeta - u_\parallel\tau)}{\langle\langle z^2(\tau)\rangle\rangle}\right)^2\right]. \tag{3.12}$$

Here, $\langle\langle z^2(\tau)\rangle\rangle$ is the second cumulant of $z(t)$, the solution of Eq. (2.2):

$$\langle\langle z^2(\tau)\rangle\rangle = \frac{2\psi(\nu\tau)\chi_\parallel}{\nu}, \tag{3.13}$$

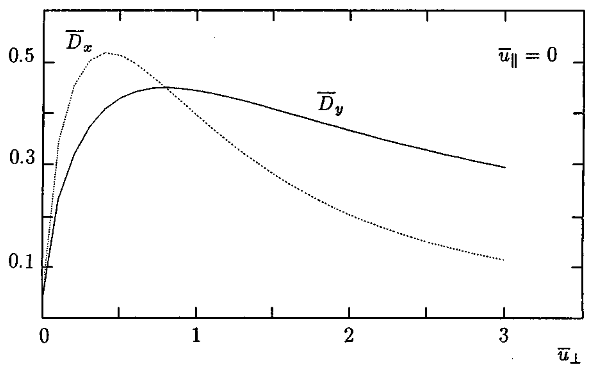

FIG. 1. The diffusion coefficients D_x and D_y resulting from Eqs. (3.14)–(3.16), as functions of $\overline{u_\perp}$ for $\overline{u_\parallel}=0$. The normalization constant is $\beta^2\chi_\parallel$ and $\gamma=2$.

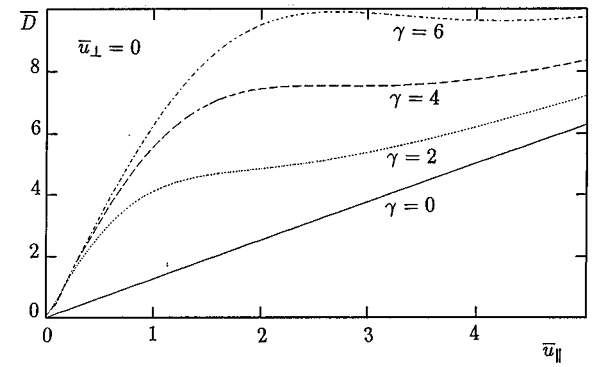

FIG. 2. The diffusion coefficient resulting from Eqs. (3.14)–(3.16) for $\overline{u_\perp}=0$ and for several values of the parameter $\gamma\equiv(\lambda_{mfp}/\lambda_\parallel)^2$. The normalization constant is $\beta^2\lambda_\parallel^2\nu$.

where $\psi(x)=x-1+\exp(-|x|)$ and $\varphi(\tau)=\frac{1}{2}d\langle\langle z^2(\tau)\rangle\rangle/d\tau=\chi_\parallel[1-\exp(-\nu\tau)]$.

Equation (3.12) shows that the stochastic parallel velocity $\eta_\parallel(t)$ has a very strong effect: the shape of the Lagrangian correlation of the product $v_i=b_i(dz/dt)$ is completely different from that of the correlation of each factor ($\mathcal{L}_{ii}^{u_\perp}$ and, respectively, $R_\parallel$). At the limit $\eta_\parallel,\chi_\parallel\to 0$ (i.e., $dz/dt=u_\parallel$) the correlation of v_i is simply $\mathcal{L}_{ii}^{u_\perp}(u_\parallel t,t)u_\parallel^2$ and has the bell-like shape of $\mathcal{L}_{ii}^{u_\perp}$. The parallel collisional velocity $\eta_\parallel$ transforms it into a more complicated function having a negative tail. When the motion is purely stochastic ($u_\parallel=0$ and $u_\perp=0$), the positive and the negative parts of $L_v^0(t)\equiv L_{v_i}(t;u_\parallel=0,\ u_\perp=0)$ have equal areas so that the diffusion coefficient is zero $[D^0=\int_0^\infty dt\ L_{v_i}^0(t)=0]$ and the mean square displacement $\Gamma(t)$ has a subdiffusive behavior. The subdiffusive behavior of $\Gamma(t)$ is thus due to collisions. They produce a random motion of the particles along magnetic lines which determine a nonzero probability density for the presence of the particles at any time moment in the correlated zone of the stochastic magnetic field. If a deterministic component of the motion exists ($u_\parallel\neq 0$ or $u_\perp\neq 0$), the particles move out of the correlated zone (they decorrelate from the magnetic lines) and $\Gamma(t)$ becomes diffusive. In mathematical terms, this process results in the fact that the deterministic drifts destroy the ''equilibrium'' between the positive and negative parts of the Lagrangian correlation $L_v^0(t)$ transforming it into a function with a positive time integral.

With the approximations (3.7) and (3.8) for $\mathcal{L}_{ii}^{u_\perp}$, Eq. (3.12) yields

$$L_{v_y}(t;u_\parallel,u_\perp)=\frac{\beta^2\lambda_\parallel}{l_\parallel}\left[R_\parallel-\frac{\varphi^2}{l_\parallel^2}+u_\parallel^2\left(1+\frac{\varphi t}{l_\parallel^2}\right)^2\right]\times\exp\left[-\frac{u_\parallel^2t^2}{2l_\parallel^2}-\frac{u_\perp^2t^2}{2\lambda_\perp^2}\right], \quad (3.14)$$

$$L_{v_x}(t;u_\parallel,u_\perp)=L_{v_y}(t;u_\parallel,u_\perp)\left[1-\frac{u_\perp^2t^2}{\lambda_\perp^2}\right], \quad (3.15)$$

where $l_\parallel^2=\lambda_\parallel^2+\langle\langle z^2\rangle\rangle$. This equation shows that both $u_\perp$ and $u_\parallel$ introduce additional time factors which strongly modify the basic correlation $L_v^0(t)$ whose ''equilibrium'' is completely destroyed. We note that the deterministic drifts have a strong effect: they determine a Gaussian decay of the correlation and also some factors which modify its shape, $u_\perp$ producing even the change of its sign. Consequently, the diffusion coefficients defined as

$$D_i(u_\parallel,u_\perp)=\int_0^\infty dt\ L_{v_i}(t;u_\parallel,u_\perp) \quad (3.16)$$

are rather complicated functions of the drift velocities.

Equations (3.14) and (3.15) show that the correlations L_{v_i} and consequently the diffusion coefficients D_x and D_y depend on three dimensionless parameters: $\overline{u_\perp}\equiv u_\perp/(\lambda_\perp\nu)$, $\overline{u_\parallel}\equiv u_\parallel/(\lambda_\parallel\nu)$, and $\gamma\equiv 2\chi_\parallel/(\lambda_\parallel^2\nu)=(\lambda_{mfp}/\lambda_\parallel)^2$. The first two parameters describe the influence of the deterministic drifts whose effect appears to be dependent on the characteristics of the two stochastic processes (collision frequency and, respectively, correlation lengths of the stochastic magnetic field). The third parameter γ describes the influence of the collisions.

The effects of the deterministic drifts on particle diffusion in a stochastic magnetic field are presented in Figs. 1–4 which show that both parallel and perpendicular deterministic drifts produce first the transition from the subdiffusive ($D=0$) to a diffusive regime and then a strong increase of the diffusion followed by a slower decay. The amplification of

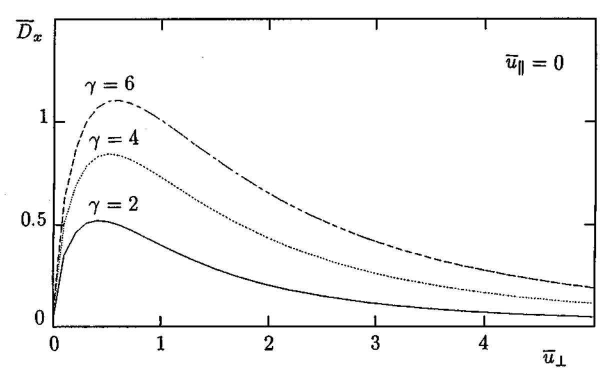

FIG. 3. The diffusion coefficient D_x (normalized with $\beta^2\lambda_\parallel^2\nu$) for $\overline{u_\parallel}=0$ and for several values of the parameter γ.

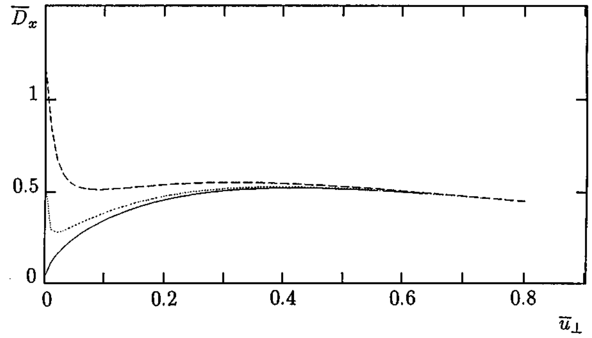

FIG. 4. The diffusion coefficient D_x in the presence of both drifts: $\overline{u_\parallel}=0.2$ (long dashed line), $\overline{u_\parallel}=0.1$ (dashed line), and $\overline{u_\parallel}=0$ (continuous line).

the diffusion is a complex effect determined by the combination of the two stochastic processes (**b** and $\eta_\parallel$) involved in particle evolution with the deterministic drifts. Figure 1 presents the dependence of the diffusion coefficient obtained from Eqs. (3.14)–(3.16) on $u_\perp$ (for $u_\parallel=0$). One can note that the perpendicular drift produces a stronger influence in the ''radial'' direction: D_x has a faster variation with $u_\perp$ than D_y.

At $u_\perp=0$, the diffusion is the same in the two directions $[D_x(u_\parallel,0)=D_y(u_\parallel,0)\equiv D(u_\parallel)]$. The effect of $u_\parallel$ is qualitatively similar to that of $u_\perp$ but it is superposed on the linear contribution which represents the diffusion in the absence of collisions (see Fig. 2). In the conditions of the tokamak plasmas, the parallel drift velocities are much smaller than the thermal velocities, $u_\parallel \ll V_T$, i.e., $\overline{u_\parallel}\ll\sqrt{\gamma}$ and thus the interesting range for $\overline{u_\parallel}$ is well before the maximum of $D(u_\parallel)$. The diffusion coefficient can be estimated analytically in this limit as

$$D(u_\parallel,0)\cong 5D_m u_\parallel \quad \text{for } \overline{u_\parallel}\ll\sqrt{\gamma}, \tag{3.17}$$

where $D_m=\sqrt{\pi/2}\beta^2\lambda_\parallel$ is the diffusion coefficient of the magnetic lines. The diffusion is thus practically independent of γ and it is five times greater than the collisionless one [which is $D^{nc}(u_\parallel)=D_m u_\parallel$, as will be checked below, Eq. (3.21)]. Figure 2 also illustrates the influence of the collisional parallel diffusivity $\chi_\parallel$ (represented by the parameter γ) on the effective diffusion coefficient: for $\gamma=0$, the well-known collisionless limit D^{nc} is recovered and, as γ increases, the nonlinear contribution determined by the combined effect of $\eta_\parallel$ and $u_\parallel$ becomes more and more important.

The stochastic collisional velocity $\eta_\parallel$ has a similar effect when $u_\parallel=0$ and $u_\perp\neq0$ (see Fig. 3): at $\gamma=0$ the diffusion coefficient is zero (the collisionless limit in this case) for any $u_\perp$. It is only when both $\eta_\parallel$ and $u_\perp$ are different from zero that a diffusion process appears and becomes stronger as γ is increasing. The diffusion coefficient can be estimated analytically from Eqs. (3.14)–(3.16) in the limit of small perpendicular drifts:

$$D_y(0,u_\perp)=D_m\left(\frac{1}{\pi}\frac{\chi_\parallel u_\perp}{\lambda_\perp}\right)^{1/2}=D_m V_T\left(\frac{\overline{u_\perp}}{2\pi}\right)^{1/2},$$

$$D_x(0,u_\perp)=1.5D_y(0,u_\perp) \quad \text{for } \overline{u_\perp}\ll\gamma. \tag{3.18}$$

Comparing Figs. 2 and 3 one can observe that a stronger amplification of the diffusion coefficient can be produced when the drift velocity is parallel to the main magnetic field.

The combined effect of the two drifts ($u_\parallel\neq0$ and $u_\perp\neq0$) is more complicated. For instance, the case $\overline{u_\parallel}\ll\sqrt{\gamma}$ is illustrated in Fig. 4. One can see that at small $\overline{u_\perp}$, $u_\parallel$ strongly modifies the effective diffusion but at large $\overline{u_\perp}$ it is practically independent of $\overline{u_\parallel}$.

The Lagrangian correlation functions (3.14) and (3.15) represent the main result of this section. They determine the mean square displacement of the particles at any time and the running diffusion coefficient (not only their asymptotic, diffusive limits). Thus it is possible to study particle behavior not only in large (infinite) size stochastic magnetic fields where the asymptotic approximation applies but also in stochastic fields with a smaller space extension. Actually, the experimental configurations belong usually to the latter case since the stochastic domains of the magnetic field appear between undestroyed magnetic surfaces or chains of islands. For such a domain having an extension along x (radial) axis of length R, the time of particle confinement t_c can be evaluated from the equation

$$\langle x^2(t_c)\rangle_{b\parallel}=R^2, \tag{3.19}$$

where $\langle x^2(t)\rangle_{b\parallel}$ is calculated according to Eq. (3.9) using the correlation function (3.15). In the special case of edge plasmas, there is a supplementary possibility of particle losses due to the fact that the stochastic magnetic lines are connected to the wall. This process determines a second equation:

$$\langle z^2(t_c)\rangle=\langle\langle z^2(t_c)\rangle\rangle+u_\parallel^2 t_c^2=Z^2(R), \tag{3.20}$$

where the length $Z(R)$ of the stochastic magnetic lines is evaluated from the diffusion equation $[2D_m Z(R)=R^2]$ and $\langle\langle z^2(t)\rangle\rangle$ is given in Eq. (3.13). The time of particle confinement in the case of edge plasmas is the minimum of the solutions of Eqs. (3.19) and (3.20). The practical method for such evaluations is to calculate $\langle x^2(t)\rangle_{b\parallel}$ and $\langle z^2(t)\rangle$ for the actual parameters of the plasma and to determine t_c graphically using the plot of these functions. Depending on the specific parameters and on the size R of the stochastic zone, various regimes are obtained: laminar or ergodic for the magnetic field, free streaming or collisional for particles. When the particles are very rapidly lost ($t_c\ll\lambda_\parallel/u_\parallel,\lambda_\perp/u_\perp$), the effect of the drifts is negligible. But the opposite condition ($t_c\geqslant\lambda_\parallel/u_\parallel,\lambda_\perp/u_\perp$) is possible in stochastic boundary layers (especially for the ions) and the drifts can modify the transport properties in this case also.

We finally note that for collisionless particles ($\eta_\parallel=0$) whose trajectories are determined by a single stochastic process **b**, the effects of the deterministic drifts are different. In this case, the Lagrangian correlations of the stochastic velocity $v_i=b_i u_\parallel$ [Eqs. (3.14) and (3.15)] reduce to $\mathcal{L}_{ii}^{u_\perp}(\zeta=u_\parallel t,t)u_\parallel^2$ and using Eq. (3.16) the following diffusion coefficients are obtained:

$$D_x^{nc} = \frac{D_m u_\parallel}{\left[1+\left(\frac{u_\perp \lambda_\parallel}{u_\parallel \lambda_\perp}\right)^2\right]^{3/2}}, \quad D_y^{nc} = \frac{D_m u_\parallel}{\left[1+\left(\frac{u_\perp \lambda_\parallel}{u_\parallel \lambda_\perp}\right)^2\right]^{1/2}}. \tag{3.21}$$

They have a different dependence on the drift velocities: a continuous increase with $u_\parallel$ and a continuous decrease with $u_\perp$.

IV. THE EFFECT OF THE COLLISIONAL CROSS FIELD DIFFUSION

In the preceding section we have studied collisional particle diffusion in stochastic magnetic fields in the presence of average drifts, neglecting the small perpendicular component $\boldsymbol{\eta}_\perp(t)$ of the stochastic velocity determined by collisions. We evaluate here the influence of $\boldsymbol{\eta}_\perp(t)$ on the effective diffusion coefficient. The problem cannot be solved using the method of Sec. III since, even in the quasilinear limit, it is not possible to express $L_{v_i}(t;u_\parallel,u_\perp)$ (the Lagrangian correlation of the velocity $v_i = b_i \eta_\parallel$ on particle trajectories) as a function of $\mathcal{L}_{ii}^{u_\perp}(\zeta,t)$ (the Lagrangian correlation of the magnetic fluctuations on magnetic lines) as was done in Eq. (3.12). This difficult (unsolved) problem can be avoided and the effective diffusion coefficient can be estimated in the quasilinear limit by using a method similar to that presented in Ref. [7] for the driftless case. This is based on the calculation of the time of collision induced decorrelation of the particles from the magnetic lines. To this aim we determine the evolution of the moments $\langle \Delta x_i \Delta x_j \rangle_{b\perp\parallel}$, $i,j=x,y$ where $\Delta x_i = x_i(t) - x_i^0(t)$ represents the contribution due to the stochastic collisional velocity $\eta_\perp$ [$x_i(t)$ is the trajectory for $\eta_\perp \neq 0$ and $x_i^0(t)$ is the trajectory for $\eta_\perp = 0$]. Straightforward calculations similar to those presented in Sec. III of Ref. [7] lead, in the quasilinear and Markovian approximation, to a set of coupled linear equations for the moments averaged over **b** and $\eta_\perp$ whose coefficients are of the type

$$\int_0^t d\tau\, \mathcal{L}_{ij}^{kl}(z(t)-z(\tau),t-\tau)\, \frac{dz}{d\tau}\frac{dz}{dt}, \quad i,j,k,l = x,y, \tag{4.1}$$

where $\mathcal{L}_{ij}^{kl}(z(t)-z(\tau),t-\tau)$ is the Lagrangian correlation of $\partial_k b_i$ and $\partial_l b_j$ on particle trajectories with $\eta_\perp = 0$ ($\partial_k b_i \equiv \partial b_i / \partial x_k$). The fact that all the Lagrangian arguments are the trajectories $x^0(t)$ allows us to estimate these correlations as in the preceding section. The Eulerian correlations of the gradients of the magnetic field can be deduced from Eq. (2.7). Then, using Corrsin factorization (3.2) one finds that the table of the correlations simplifies considerably in the Lagrangian frame: part of them are identically zero and the others can be expressed in terms of three functions:

$$\mathcal{L}_{xx}^{xx} = \mathcal{L}_{yy}^{yy} = -\mathcal{L}_{xy}^{xy} = -\mathcal{L}_{yx}^{xy} = \mathcal{L}_1(\zeta,t), \tag{4.2}$$

$$\mathcal{L}_{xx}^{xy} = \mathcal{L}_{yx}^{yy} = \mathcal{L}_{yy}^{xy} = \mathcal{L}_{xy}^{xx} = \mathcal{L}_{ij}^{0l} = 0, \tag{4.3}$$

$$\mathcal{L}_{yy}^{xx} = \mathcal{L}_2(\zeta,t), \tag{4.4}$$

$$\mathcal{L}_{xx}^{yy} = \mathcal{L}_3(\zeta,t), \tag{4.5}$$

where the index 0 in Eq. (4.3) indicates the correlation of b_i with $\partial_l b_j$. The functions $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ are given by

$$\mathcal{L}_1(\zeta,t) = \beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right) \frac{\lambda_\perp^4 (J_y - u_\perp^2 t^2)}{J_x^{3/2} J_y^{5/2}} \exp\left(-\frac{u_\perp^2 t^2}{2J_y}\right), \tag{4.6}$$

$$\mathcal{L}_2(\zeta,t) = 3\beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right) \frac{\lambda_\perp^4}{J_x^{5/2} J_y^{1/2}} \exp\left(-\frac{u_\perp^2 t^2}{2J_y}\right), \tag{4.7}$$

$$\mathcal{L}_3(\zeta,t) = \beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right) \frac{\lambda_\perp^4 (3J_y^2 - 6u_\perp^2 t^2 J_y + u_\perp^4 t^4)}{J_x^{1/2} J_y^{9/2}} \times \exp\left(-\frac{u_\perp^2 t^2}{2J_y}\right), \tag{4.8}$$

where J_i have the same definition as in Eqs. (3.5) and (3.6). In the quasilinear limit ($\alpha \equiv \beta\lambda_\parallel/\lambda_\perp \ll 1$), $J_i \approx \lambda_\perp^2$ and Eqs. (4.6)–(4.8) reduce to much simpler expressions:

$$\mathcal{L}_1(\zeta,t) \cong \frac{\beta^2}{\lambda_\perp^2} \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2} - \frac{u_\perp^2 t^2}{2\lambda_\perp^2}\right)\left(1 - \frac{u_\perp^2 t^2}{\lambda_\perp^2}\right), \tag{4.9}$$

$$\mathcal{L}_2(\zeta,t) \cong 3\frac{\beta^2}{\lambda_\perp^2} \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2} - \frac{u_\perp^2 t^2}{2\lambda_\perp^2}\right), \tag{4.10}$$

$$\mathcal{L}_3(\zeta,t) \cong 3\frac{\beta^2}{\lambda_\perp^2} \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2} - \frac{u_\perp^2 t^2}{2\lambda_\perp^2}\right)\left(1 - 2\frac{u_\perp^2 t^2}{\lambda_\perp^2} + \frac{u_\perp^4 t^4}{3\lambda_\perp^4}\right). \tag{4.11}$$

The set of equations for the moments becomes

$$\frac{d}{dt}\langle \Delta x^2 \rangle_{b\perp\parallel} = 2A_1 \langle \Delta x^2 \rangle_{b\perp\parallel} + 2A_3 \langle \Delta y^2 \rangle_{b\perp\parallel} + 2\chi_\perp(t), \tag{4.12}$$

$$\frac{d}{dt}\langle \Delta y^2 \rangle_{b\perp\parallel} = 2A_2 \langle \Delta x^2 \rangle_{b\perp\parallel} + 2A_1 \langle \Delta y^2 \rangle_{b\perp\parallel} + 2\chi_\perp(t), \tag{4.13}$$

$$\frac{d}{dt}\langle \Delta x \Delta y \rangle_{b\perp\parallel} = -4A_1 \langle \Delta x \Delta y \rangle_{b\perp\parallel}, \tag{4.14}$$

where $A_i(t) = \int_0^t d\tau\, \mathcal{L}_i(z(t)-z(\tau),t-\tau)(dz/d\tau)(dz/dt)$ and $\chi_\perp(t) = \int_0^t d\tau\, R_\perp(\tau)$. With the initial condition $\Delta x_i(0) = 0$, the solution of Eqs. (4.12)–(4.14) is

$$\langle \Delta x^2(t) \rangle_{b\perp} = \left[1 + \left(\frac{A_3}{A_2}\right)^{1/2}\right] \int_0^t dt_1 \chi_\perp(t_1) \times \exp\left[2\int_{t_1}^t (A_1 + \sqrt{A_2 A_3})\, d\theta\right], \tag{4.15}$$

$$\langle \Delta y^2(t) \rangle_{b\perp} = \left[1 + \left(\frac{A_2}{A_3}\right)^{1/2}\right] \int_0^t dt_1 \chi_\perp(t_1) \times \exp\left[2\int_{t_1}^t (A_1 + \sqrt{A_2 A_3})\, d\theta\right], \tag{4.16}$$

$$\langle \Delta x(t)\Delta y(t)\rangle_{b\perp\parallel}=0, \tag{4.17}$$

where we have retained, for simplicity, only the fastest growing terms. This solution is valid for small time, so that $\langle \Delta x_i^2(t)\rangle_{b\perp\parallel}\ll\lambda_\perp^2$.

The relative trajectory dispersion [Eqs. (4.15) and (4.16)] has still to be averaged over the stochastic collisional velocity $\eta_\parallel$. Due to the complicated dependence on $\eta_\parallel$ and $z(t)$ appearing in Eqs. (4.15) and (4.16), it is not possible to calculate this average, except in the limit of small perpendicular drift velocity $u_\perp$ where we can neglect the anisotropy introduced by $u_\perp$ and approximate $A_2\cong A_3\cong 3A_1$. In this case

$$\langle \Delta x^2(t)\rangle_{b\perp}\cong\langle \Delta y^2(t)\rangle_{b\perp} \cong 2\int_0^t dt_1 \chi_\perp(t_1)\exp\left(8\int_{t_1}^t A_1(\theta)d\theta\right). \tag{4.18}$$

With this approximation, the calculations follow the same steps as in our previous study [20] concerning collision induced decorrelation in a time-dependent stochastic magnetic field. $\eta_\parallel$ is contained only in the argument of the exponential in Eq. (4.18) and the average over $\eta_\parallel$ can be calculated using the cumulant method (second cumulant). The first cumulant is estimated as

$$C_1(u_\parallel,u_\perp;t,t_1)=\frac{8}{\lambda_\perp^2}\int_{t_1}^t d\theta\int_0^{t_1}d\theta_1 L_{v_x}(\theta-\theta_1;u_\parallel,u_\perp) \cong\frac{8D_x^0(u_\parallel,u_\perp)}{\lambda_\perp^2}(t-t_1) \tag{4.19}$$

using Eqs. (4.9), (3.10), (3.15), and the asymptotic (large t,t_1) approximation. Here, $D_x^0(u_\parallel,u_\perp)$ is the drift induced diffusion coefficient for $\boldsymbol{\eta}_\perp=\mathbf{0}$ (determined in the preceding section). Long but straightforward calculations showed that the drifts determine the decay of the second cumulant which is smaller than the first except in the limit $u_\perp\cong 0$ and $u_\parallel\cong 0$ where it is dominant and can be approximated by

$$C_2(u_\parallel=0,u_\perp=0;t,t_1) \cong 32(\pi-2)\beta^4\,\frac{\lambda_\parallel^2[\langle\langle z^2(t)\rangle\rangle-\langle\langle z^2(t_1)\rangle\rangle]}{\lambda_\perp^4}. \tag{4.20}$$

The average over $\eta_\parallel$ of the exponential in Eq. (4.18) becomes

$$\left\langle \exp\left[8\int_{t_1}^t d\theta\int_0^{t_1}d\theta_1\mathcal{L}_i(z(\theta)-z(\theta_1),\theta-\theta_1)\,\frac{dz}{d\theta}\,\frac{dz}{d\theta_1}\right]\right\rangle_\parallel \cong\exp\left[\frac{8a}{\lambda_\perp^2}(t-t_1)\right], \tag{4.21}$$

where $a=D_x^0(u_\parallel,u_\perp)+4(\pi-2)\beta^2\alpha^2\chi_\parallel$. Finally, we have obtained the following evaluation of the average over $\eta_\parallel$ of Eq. (4.18) (see Ref. [20]):

$$\langle \Delta x^2(t)\rangle_{b\perp\parallel}\cong\begin{cases}\dfrac{8a\chi_\perp}{\lambda_\perp^2}\,t^2, & \text{KP regime}\\[2mm] \dfrac{\lambda_\perp^2\chi_\perp}{4a}\exp\left(\dfrac{8a}{\lambda_\perp^2}\,t\right), & \text{RR regime}\end{cases} \tag{4.22}$$

where the first line was obtained for $L_{\rm KP}\equiv\lambda_\perp\sqrt{\chi_\parallel/\chi_\perp}\ll L_K$ which corresponds to the Kadomtsev-Pogutse collisional regime and the second line for $L_{\rm KP}\gg L_K$, i.e., for the Rechester-Rosenbluth regime [$L_K\equiv\lambda_\perp^2/(4\sqrt{\pi/2}\beta^2\lambda_\parallel)$ is the exponentiation length of the magnetic lines]. A detailed classification of the collisional regimes of particle diffusion in stochastic magnetic fields is presented in Ref. [11].

The dispersion of the trajectories can be written as the sum

$$\langle x^2(t)\rangle_{b\perp\parallel}=\langle [x^0(t)]^2\rangle_{b\parallel}+\langle \Delta x^2(t)\rangle_{b\perp\parallel}, \tag{4.23}$$

where the first term contains the effect of the drifts for $\boldsymbol{\eta}_\perp=\mathbf{0}$ and the second the influence of the collisional velocity $\boldsymbol{\eta}_\perp$. This decomposition induces a similar relationship for the diffusion coefficient: $D_i=D_i^0+D_i^c$ where D_i^c is the contribution of the perpendicular collisional diffusivity. The diffusion coefficient can be estimated as $D_i\approx\lambda_\perp^2/(2t_d)$ where the decorrelation time t_d is determined from the equation

$$\langle [x^0(t_d)]^2\rangle_{b\parallel}+\langle \Delta x^2(t_d)\rangle_{b\perp\parallel}=\lambda_\perp^2. \tag{4.24}$$

Straightforward analysis of this equation shows that for small drifts the second term of Eq. (4.24) is dominant and the decorrelation is produced mainly by collisions. The decorrelation time t_d is estimated from $\langle \Delta x^2(t_d)\rangle_{b\perp\parallel}=\lambda_\perp^2$ and the random walk evaluation of the diffusion coefficient yields

$$D^c\cong\begin{cases}\sqrt{2[4(\pi-2)\beta^2\alpha^2\chi_\parallel+D^0]\chi_\perp}+D^0, & \text{KP regime}\\[2mm] 4\,\dfrac{4(\pi-2)\beta^2\alpha^2\chi_\parallel+D^0}{\ln\left[4\,\dfrac{4(\pi-2)\beta^2\alpha^2\chi_\parallel}{\chi_\perp}\right]}+D^0, & \text{RR regime.}\end{cases} \tag{4.25}$$

For such small drifts $D^0(u_\parallel,u_\perp)$ is small and it represents in Eq. (4.25) a small correction to the driftless value which is recovered at $u_\parallel=u_\perp=0$ as the well-known Kadomtsev-Pogutse or Rechester-Rosenbluth diffusion coefficient. Equation (4.25) is valid for small drift velocities such that the first term in Eq. (4.24) can be neglected, i.e., for $2D^0t_d\ll\lambda_\perp^2$. This condition reads

$$D^0(u_\parallel,u_\perp)\ll D^0_{\rm lim}=\begin{cases}2\chi_\perp, & \text{KP regime}\\ 4(\pi-2)\beta^2\alpha^2\chi_\parallel, & \text{RR regime.}\end{cases} \tag{4.26}$$

It corresponds to perpendicular drift velocities:

$$\overline{u_\perp}\ll\overline{u_\perp^{\rm lim}}=\begin{cases}4\left(\dfrac{\rho_L}{\beta\lambda_\parallel}\right)^4\dfrac{1}{\gamma}, & \text{KP regime}\\[2mm] 16\alpha^4\gamma, & \text{RR regime}\end{cases} \tag{4.27}$$

or alternatively to parallel drift velocities:

$$\overline{u_\parallel}\ll\overline{u_\parallel^{\rm lim}}=\begin{cases}\dfrac{1}{5}\left(\dfrac{\rho_L}{\beta\lambda_\parallel}\right)^2, & \text{KP regime}\\[2mm] \dfrac{2}{5}\,\alpha^2\gamma, & \text{RR regime.}\end{cases} \tag{4.28}$$

Equations (3.17) and (3.8) were used for the estimates (4.27) and (4.28) and thus they are valid only if $\overline{u}_\perp^{\text{lim}}$ and $\overline{u}_\parallel^{\text{lim}}$ are in the validity range for those estimates ($\overline{u}_\perp^{\text{lim}} \ll \gamma$ and $\overline{u}_\parallel^{\text{lim}} \ll \sqrt{\gamma}$).

At larger drift velocities when $D_i^0 \gg D_{\text{lim}}^0$, the first term of Eq. (4.24) becomes dominant compared to the second one and the influence of the perpendicular collisional diffusivity $\chi_\perp$ on the effective diffusion coefficient is negligible. Consequently, the effective diffusion coefficient can be approximated by the result obtained in the preceding section:

$$D_i(u_\perp, u_\parallel) \cong D_i^0(u_\perp, u_\parallel). \qquad (4.29)$$

In conclusion, we have shown that the perpendicular component of the collisional stochastic velocity $\boldsymbol{\eta}_\perp$ is important only in the limit of small drifts [when the effective diffusion coefficient is represented by Eq. (4.25)]. Consequently, the Kadomtsev-Pogutse or Rechester-Rosenbluth diffusion coefficient is a good approximation for the effective diffusion only in the limit of small drifts. At higher drift velocities such that $D_i^0 \gg D_{\text{lim}}^0$, the effective diffusion coefficient Eq. (4.29) is practically independent of the collisional cross field diffusivity $\chi_\perp$ and it is completely different from the driftless one.

V. CONCLUSION

We have shown here that the presence of plasma flows can determine a strong effect on collisional particle diffusion in a stochastic magnetic field. It consists in a large amplification of the diffusion coefficients. As an example, for the usual experimental parameters of the tokamak plasma (electron temperature of 1000 eV, density of 3×10^{19} m^{-3}, $B_0 \cong 3$ T, $\beta \cong 10^{-4}$, $\lambda_\parallel \cong 1$ m, $\lambda_\perp \cong 10^{-2}$ m) a very small value of the magnetic diffusion coefficient is obtained ($D_m = 10^{-8}$ m). The parallel drift corresponding to the toroidal current (with a typical density of 10^6 A/m^2) has only a weak effect in these conditions but it can become important at smaller densities. However, a radial electric field of 2 kV/m determines a perpendicular velocity, which leads, according to Eq. (3.18), to a rather strong diffusion coefficient of the order 0.2 m^2/sec.

We have also shown that the cross field collisional diffusivity has an important effect on the effective particle diffusion only in the limit of small deterministic drifts where two regimes for the diffusion coefficient are observed according to the ratio L_{KP}/L_K, Eq. (4.25). They are, up to small corrections determined by the average drifts, the well-known Kadomtsev-Pogutse and Rechester-Rosenbluth diffusion coefficients. When the deterministic drifts are increased both regimes evolve to the same diffusion coefficient which is independent of $\chi_\perp$. In particular, a plasma in the Kadomtsev-Pogutse regime which is characterized by a stochastic field diffusion coefficient smaller than the collisional one could be, in fact, in a highly anomalous regime if appropriate deterministic drifts are present. When the drifts are large enough the Kadomtsev-Pogutse or Rechester-Rosenbluth diffusion coefficients are not valid and must be replaced by the drift induced diffusion coefficient $D_i^0(u_\perp, u_\parallel)$ [Eqs. (3.14)–(3.16) or the approximations (3.17), (3.18)]. For the example considered in the preceding paragraph, the Rechester-Rosenbluth model yields a diffusion coefficient of the order 0.01 m^2/sec which is 20 times smaller than the perpendicular drift induced diffusion coefficient.

The analytical expressions (3.14) and (3.15) obtained for the Lagrangian correlation of the stochastic velocity contain all the transitory regimes which characterize both the stochastic magnetic field and the collisional velocity of the particles. Consequently, it can be used for studying finite stochastic regions and also stochastic boundaries such as ergodic divertors in tokamak plasmas.

ACKNOWLEDGMENTS

M.V. and F.S. are grateful for the warm hospitality of the members of the Department of Controlled Fusion Research of CE-Cadarache and of the Department of Statistical Physics and Plasmas of the Université Libre de Bruxelles. They acknowledge the financial support for their stay at CE-Cadarache from the ''Commissariat à l'Energie Atomique'' and from the ''Ministère des Affaires Etrangères de France.'' This work was partly prepared during the stay of M.V. and F.S. at Université Libre de Bruxelles where they have been supported financially by the Association Euratom-Belgian State for Fusion and by a grant from the European Communities in the frame of the PECO Program.

[1] A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).
[2] B. B. Kadomtsev and O. P. Pogutse, in *Plasma Physics and Controlled Nuclear Fusion Research, Proceedings of the 7th International Conference, Innsbruck 1978* (International Atomic Energy Agency, Vienna, 1979), Vol. 1, p. 649.
[3] J. H. Krommes, C. Oberman, and R. G. Kleva, J. Plasma Phys. **30**, 11 (1983).
[4] A. Thyagaraja and F. Haas, Phys. Scr. **31**, 83 (1985).
[5] M. B. Isichenko, Plasma Phys. Controlled Fusion **33**, 795 (1991).
[6] L. Hannibal, Phys. Fluids **B5**, 3551 (1993).
[7] Hai-Da Wang, M. Vlad, E. Vanden Eijnden, F. Spineanu, J. H. Misguich, and R. Balescu, Phys. Rev. E **51**, 4844 (1995).
[8] H. Sugimoto, T. Kurasawa, and H. Ashida, Plasma Phys. Controlled Fusion **36**, 383 (1994).
[9] G. Laval, Phys. Fluids **B5**, 711 (1993).
[10] R. Balescu, Hai-Da Wang, and J. H. Misguich, Phys. Plasmas **1**, 3826 (1994).
[11] J. H. Misguich, M. Vlad, F. Spineanu, and R. Balescu, Comments Plasma Phys. Controlled Fusion **17**, 45 (1995).
[12] F. Spineanu and M. Vlad, J. Plasma Phys. **54**, 333 (1995).
[13] J. M. Rax and R. B. White, Phys. Rev. Lett. **68**, 1523 (1992).
[14] H. E. Mynick and J. A. Krommes, Phys. Rev. Lett. **43**, 1506 (1979); Phys. Fluids **23**, 1229 (1980).
[15] J. R. Myra and P. Catto, Phys. Fluids **B4**, 176 (1992).
[16] J. R. Myra, P. Catto, H. E. Mynick, and R. E. Duvall, Phys. Fluids **B5**, 1160 (1993).

[17] M. Coronado, E. J. Vitela, and A. Akcasu, Phys. Fluids **B4**, 3935 (1992).

[18] M. B. Isichenko and P. H. Diamond, University of Texas, Austin Fusion Research Center Report No. FRCR 451, 1994 (unpublished).

[19] H. Capes, A. Samain, F. Nguyen, and Ph. Ghendrih, in *Stellarators and Other Helical Confinement Systems, Proceedings of the International Atomic Energy Agency Technical Committee Meeting, Garching, 1993* (International Atomic Energy, Vienna, 1993), Vol. 2, p. 516.

[20] M. Vlad, F. Spineanu, J. H. Misguich, and R. Balescu, Phys. Rev. E **53**, 5302 (1996).

[21] W. D. McComb, *The Physics of Fluid Turbulence* (Clarendon, Oxford, 1990).